High School—Algebra II

In Algebra II, <u>a one-credit course</u>, students build on their work with linear, quadratic, and exponential functions, to extend their repertoire of functions to include polynomial, rational, and radical functions. Students work closely with the expressions that define the functions, and continue to expand and hone their abilities to model situations and to solve equations, including solving quadratic equations over the set of complex numbers and solving exponential equations using the properties of logarithms. The Mathematical Practice Standards apply throughout this course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations. The four critical areas of this course include (1) working extensively with polynomial operations; (2) building connections between geometry and trigonometric ratios; (3) understanding of a variety of function families; and (4) explore statistical data. Each critical area is described below:

> (1) Students develop the structural similarities between the system of polynomials and the system of integers. Students draw on analogies between polynomial arithmetic and base-ten computation, focusing on properties of operations, particularly the distributive property. Students connect multiplication of polynomials with multiplication of multi-digit integers, and division of polynomials with long division of integers. Students identify zeros of polynomials, including complex zeros of quadratic polynomials, and make connections between zeros of polynomials and solutions of polynomial equations. The unit culminates with the fundamental theorem of algebra. A central theme of this unit is that the arithmetic of rational expressions is governed by the same rules as the arithmetic of rational numbers.

> (2) Building on their previous work with functions, and on their work with trigonometric ratios and circles in Geometry, students now use the coordinate plane to extend trigonometry to model periodic phenomena.

> (3) Students synthesize and generalize what they have learned about a variety of function families. They extend their work with exponential functions to include solving exponential equations with logarithms. They explore the effects of transformations on graphs of diverse functions, including functions arising in an application, in order to abstract the general principle that transformations on a graph always have the same effect regardless of the type of the underlying function. They identify appropriate types of functions to model a situation, they adjust parameters to improve the model, and they compare models by analyzing appropriateness of fit and making judgments about the domain over which a model is a good fit. The description of modeling as "*the process of choosing and using mathematics and statistics to analyze empirical situations, to understand them better, and to make decisions*" is at the heart of this unit. The narrative discussion and diagram of the modeling cycle should be considered when knowledge of functions, statistics, and geometry is applied in a modeling context.

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(4) Students see how the visual displays and summary statistics they learned in earlier grades relate to different types of data and to probability distributions. They identify different ways of collecting data—including sample surveys, experiments, and simulations—and the role that randomness and careful design play in the conclusions that can be drawn

The content of this document is centered on the mathematics domains of **Counting and Cardinality** (Grade K), **Operations and Algebraic Thinking; Numbers and Operations in Base Ten** (Grades K-5); **Numbers and Operations—Fractions** (Grades 3-5); **Measurement and Data** (Grades K-5); **Ratios and Proportional Relationships** (Grades 6-7); **the Number System, Expressions & Equations, Geometry, Statistics & Probability** (Grades 6-8); **Functions** (Grade 8), and the high school conceptual categories of **Number and Quantity**, **Algebra**, **Functions**, **Modeling, Geometry**, and **Statistics & Probability**. Instruction in these domains and conceptual categories should be designed to expose students to experiences, which reflect the value of mathematics, to enhance students' confidence in their ability to do mathematics, and to help students communicate and reason mathematically.

Algebra II
Number and Quantity

The Real Number System (N-RN)

Extend the properties of exponents to rational exponents

N-RN.1	Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define $5^{1/3}$ to be the cube root of 5 because we want $[5^{1/3}]^3 = 5^{(1/3)\,3}$ to hold, so $[5^{1/3}]^3$ must equal 5.*
N-RN.2	Rewrite expressions involving radicals and rational exponents using the properties of exponents.

Quantities (N-Q) *

Reason quantitatively and use units to solve problems

N-Q.2	Define appropriate quantities for the purpose of descriptive modeling.*

The Complex Number System (N-CN)

Perform arithmetic operations with complex numbers

N-CN.1	Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.
N-CN.2	Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

Use complex numbers in polynomial identities and equations

N-CN.7	Solve quadratic equations with real coefficients that have complex solutions.

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Seeing Structure in Expressions (A-SSE)

Interpret the structure of expressions

A-SSE.2	Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.*

Write expressions in equivalent forms to solve problems

A-SSE.3	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.* c. Use the properties of exponents to transform expressions for exponential functions. *For example the expression 1.15t can be rewritten as $[1.15^{1/12}]^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*

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A-SSE.4	Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.**
Arithmetic with Polynomials and Rational Expressions (A-APR)	
Understand the relationship between zeros and factors of polynomials	
A-APR.2	Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.
A-APR.3	Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial (limit to 1st- and 2nd-degree polynomials).
Use polynomial identities to solve problems	
A-APR.4	Prove polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples.*
Rewrite rational expressions	
A-APR.6	Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.
Creating Equations (A-CED) *	
Create equations that describe numbers or relationships	
A-CED.1	Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.**
A-CED.2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. *[Note this standard appears in previous courses with a slight variation in the standard language.]*
	Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.
Reasoning with Equations and Inequalities (A-REI)	
Understand solving equations as a process of reasoning and explain the reasoning	
A-REI.1	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
A-REI.2	Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

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	Solve equations and inequalities in one variable
A-REI.4	Solve quadratic equations in one variable. b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions.

	Solve systems of equations
A-REI.6	Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.
A-REI.7	Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line y = -3x and the circle $x^2 + y^2 = 3$.*

	Represent and solve equations and inequalities graphically
A-REI.11	Explain why the *x*-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.*

Functions

Interpreting Functions (F-IF)

	Understand the concept of a function and use function notation
F-IF.3	Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) for n ≥ 1.*

	Interpret functions that arise in applications in terms of the context
F-IF.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*
F-IF.6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.*

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Analyze functions using different representations	

F-IF.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.*
	   c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
	   e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

F-IF.8	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as y = (1.02)t, y = (0.97)t, y = (1.01)12t, y = (1.2)t/10, and classify them as representing exponential growth and decay.
F-IF.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

	Building Functions (F-BF)

Build a function that models a relationship between two quantities	

F-BF.1	Write a function that describes a relationship between two quantities.*
	   a. Determine an explicit expression, a recursive process, or steps for calculation from a context.
	   b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*

F-BF.2	Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.*

Build new functions from existing functions	
F-BF.3	Identify the effect on the graph of replacing *f(x)* by *f(x)* + *k*, *k f(x)*, *f(kx)*, and *f(x + k)* for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*
F-BF.4	Find inverse functions. a. Solve an equation of the form *f(x)* = *c* for a simple function *f* that has an inverse and write an expression for the inverse. *For example, f(x) =2x³ or f(x) = (x+1)/(x-1) for x ≠ 1.*

	Linear, Quadratic, and Exponential Models (F-LE) *

Construct and compare linear, quadratic, and exponential models and solve problems	

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F-LE.2	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).*
F-LE.3	Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.*
F-LE.4	For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.*
	Interpret expressions for functions in terms of the situation they model
F-LE.5	Interpret the parameters in a linear or exponential function in terms of a context.*
	Trigonometric Functions (F-TF)
	Extend the domain of trigonometric functions using the unit circle
F-TF.1	Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
F-TF.2	Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

Geometry

	Expressing Geometric Properties with Equations (G-GPE)
	Translate between the geometric description and the equation for a conic section
G-GPE.2	Derive the equation of a parabola given a focus and directrix.

Statistics and Probability*

	Interpreting Categorical and Quantitative Data (S-ID)
	Summarize, represent, and interpret data on a single count or measurement variable
S-ID.4	Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.*
	Summarize, represent, and interpret data on two categorical and quantitative variables
S-ID.6	Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.* a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.*
	Making Inferences and Justifying Conclusions (S-IC)

	Understand and evaluate random processes underlying statistical experiments
S-IC.1	Understand statistics as a process for making inferences about population parameters based on a random sample from that population.*

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S-IC.2	Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?**
	Make inferences and justify conclusions from sample surveys, experiments, and observational studies
S-IC.3	Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.*
S-IC.4	Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.*
S-IC.5	Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.*
S-IC.6	Evaluate reports based on data.*
Conditional Probability and the Rules of Probability (S-CP)	
	Understand independence and conditional probability and use them to interpret data
S-CP.1	Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").*
S-CP.2	Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.*
S-CP.3	Understand the conditional probability of A given B as $P(A$ and $B)/P(B)$, and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.*
S-CP.4	Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.**
S-CP.5	Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.**

	Use the rules of probability to compute probabilities of compound events in a uniform probability model
S-CP.6	Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.*
S-CP.7	Apply the Addition Rule, $P(A$ or $B) = P(A) + P(B) - P(A$ and $B)$, and interpret the answer in terms of the model.*

* Modeling Standards

Additional Resource

2016 Mississippi College- and Career-Standards Scaffolding Document

The primary purpose of the *2016 Mississippi College- and Career-Readiness Standards Scaffolding Document* is to provide teachers with a deeper understanding of the Standards as they plan for classroom instruction. Based on the 2016 Mississippi College- and Career-Readiness Standards for Mathematics, this document provides a close analysis of the requirements for student mastery. Because of the rigor and depth of the Standards, scaffolding instruction to meet the needs of all learners is essential to individual success. The Scaffolding Document will aid teachers' understanding of how to teach the Standards through a natural progression of student mastery. The Scaffolding Document can be found at **http://www.mde.k12.ms.us/ESE/ccr**.

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.

2. Reason abstractly and quantitatively.

3. Construct viable arguments and critique the reasoning of others.

4. Model with mathematics.

5. Use appropriate tools strategically.

6. Attend to precision.

7. Look for and make use of structure.

8. Look for and express regularity in repeated reasoning.